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Painting in polarization

Aaron D. Slepko^{a)}

Physics & Astronomy, Trent University, Peterborough, Ontario K9L 0G2, Canada

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The array of colors and patterns seen when birefringent materials are inserted between polarizers is a source of amusement, popular science demonstrations, and art. This phenomenon of polarization-filtered coloration is commonly but misleadingly referred to as “interference colors,” despite not arising from the effects of interference. In this work, I clarify the link between polarization filtering and the observed colors and demonstrate how various aspects of birefringence in common household films provide opportunities and challenges for their use in art. © 2022 Published under an exclusive license by American Association of Physics Teachers.

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I. INTRODUCTION

The array of patterned colors that can arise when birefringent materials are sandwiched between linear polarizers has been appreciated both for beauty and usefulness for as long as polarized light has been produced.^{1–3} In fact, it has become commonplace for optics textbooks to include photographs of colorful arrangements of cellophane tape, stressed glass, or plastic utensils as an introductory visual illustration of birefringence.⁴ Beyond aesthetic attributes, the phenomenon has found long-standing use in various technical fields. For example, minute anisotropies in annealed glass can give rise to local “stress-induced” birefringence that can be visualized in polarized light,⁵ and glassblowers commonly use the technique to reveal unseen internal stress near cooled joints. Since the early 1830s, geologists and mineralogists have developed a framework for using the patterns and colors that arise in a polarization microscope to identify mineral samples.⁶ The Michel-Lévy interference color chart is widely used by geologists to match observed color to sample thickness, orientation, and birefringence.⁷

Inspired both by challenge and opportunity, artists have sought to move beyond random patterning and coloration displayed by everyday birefringent objects to create deliberately constructed vividly colored images using this phenomenon. Undoubtedly, the artist best known for this is Austine Wood Comarow (1942–2020).⁸ Austine pioneered the art of the polarization collage, or “polage,” as it has come to be known.⁹ Using sophisticated layering of cut cellophane and other birefringent polymer films, interspersed with layers of film polarizers, Austine created a wide array of works, ranging from small standalone pieces that fit on a shelf, to massive career-spanning installations in institutions such as the Disney Epcot Center in Florida (in 1981) and the Gyeongsangnam-do Institute of Science Education in Korea (in 2017).¹⁰ Austine’s early attempts at deterministic localized control of birefringence-borne colors presaged full-color liquid crystal display (LCD) technology. Indeed, the careful control of electrically induced birefringence in microscopic elements of liquid crystals sandwiched between crossed-polarizers is the principal operating mechanism for the LCD displays in computer monitors, laptops, tablets, and cellphones.¹¹

This paper aims to clarify the underlying physical process that gives rise to colors seen in birefringent samples when observed through polarizers, especially in the context of art and educational demonstrations. Through examples that

incorporate common everyday polarization and birefringent materials, I provide insight into the basic physical attributes of these colored structures, including the dramatic effects of thickness variation on perceived colors and the non-intuitive effects of layer addition. This updated and physically minded review of the phenomenon should prove useful to science educators, students, and artists alike, providing a foundation for understanding the observed colors from a spectral standpoint.

II. BIREFRINGENCE, RETARDANCE, AND THE POLARIZATION GATE

Birefringence is an optical property wherein the index of refraction depends on the polarization directions of light. Whereas isotropic materials present a uniform index of refraction to incident light of all polarizations, crystalline or otherwise structured materials may present two or three distinct indexes of refraction along specific optic axes. Quasi-two-dimensional birefringent materials, such as the polymer films described in this work, display uniaxial birefringence, meaning that there are two distinct orthogonal directions (optic axes) corresponding to the maximum and minimum index of refraction. At its most basic, the index of refraction affects the propagation speed of light. The index of refraction is determined by the projection of the electric field onto the optic axes. When the light is polarized along one of the optic axes in the material, all the light propagates with a single index, and its polarization state is, thus, unaffected by the existence of other directions with different indexes of refraction; the light does not experience any birefringence. On the other hand, light that is polarized at an angle to an optic axis can be broken down (mathematically or conceptually) into orthogonal components projected onto the optic axes, with each component traveling at a different speed in the material. Quantitatively, we can define the birefringence of the sample as the difference between the maximum and minimum index of refraction, Δn .

As different components of the electric field experience different indexes of refraction, they travel at slightly different speeds. Any such mismatch in speeds can be described in terms of a temporal delay, or in terms of an evolving retardance between phase components of the light. Phase retardance, δ , expresses the difference in phase (in units of radians) between the electric field component that is along the optic axis and the component that is perpendicular to it. For a birefringent material of thickness d , under normal

incidence, the retardance depends on the wavelength of the light (λ) and on the magnitude of birefringence such that

$$\delta = \frac{2\pi d \Delta n}{\lambda}. \quad (1)$$

It is the combination of this phase retardance and the angle between the incoming polarization and the optic axis, θ , that determines the polarization state of the light as it emerges from the birefringent material. Incident linearly polarized light may emerge from the birefringent material either unaffected, linearly polarized along a new direction, elliptically polarized with rotated (or the same) major axis, or circularly polarized. Furthermore, the dependence of retardance on the wavelength of light means that, for a given sample orientation, birefringence, and thickness, the polarization state of the light emerging from the birefringent layer is continuously changing across the spectrum. However, since neither our eyes nor most cameras are sensitive to polarization state, we are not able to discern any visual difference in color between the incident light and the light emerging from a given birefringent sample submitted to white-light illumination.

Observation of birefringence-dependent colors requires a transparent sample with regions of retardance to be placed between an initial polarizer (which polarizes the input light) and a second polarizer (which is called the analyzer). Together, the polarizer and analyzer comprise what is known as a polarization gate. If the analyzer's polarization axis is aligned with that of the entrance polarizer, the condition is referred to as an open gate, and if the analyzer is

perpendicular to the entrance polarizer, the condition is referred to as a closed gate. The analyzer transmits an intensity that depends on the polarization state of the light emerging from the sample and its orientation relative to the analyzer's polarization axis. A polarizer fully transmits polarization components aligned with it, fully extinguishes components that are perpendicular to its axis, and attenuates all intermediate polarizations. Thus, the different polarizations that correspond to each wavelength are transmitted in different proportions, thereby resulting in a variable spectral intensity that we observe as distinct colors. This process is schematically illustrated in Fig. 1.

In general, the transmitted spectrum, $I(\lambda)$, as a function of the incident spectrum, $I_0(\lambda)$, and the angle between the input polarizer axis and the sample's optic axis, θ , are given by¹²

$$I_{CG}(\lambda) = I_0(\lambda) \sin^2\left(\frac{\pi}{\lambda} d \Delta n\right) \sin^2(2\theta), \quad (2)$$

and the transmission coefficient can be expressed as

$$T_{CG}(\lambda) \equiv \frac{I_{CG}(\lambda)}{I_0(\lambda)} = \sin^2\left(\frac{\delta}{2}\right) \sin^2(2\theta). \quad (3)$$

For the open-gate configuration

$$I_{OG}(\lambda) = I_0(\lambda) \left[1 - \sin^2\left(\frac{\pi}{\lambda} d \Delta n\right) \sin^2(2\theta) \right] \quad (4)$$

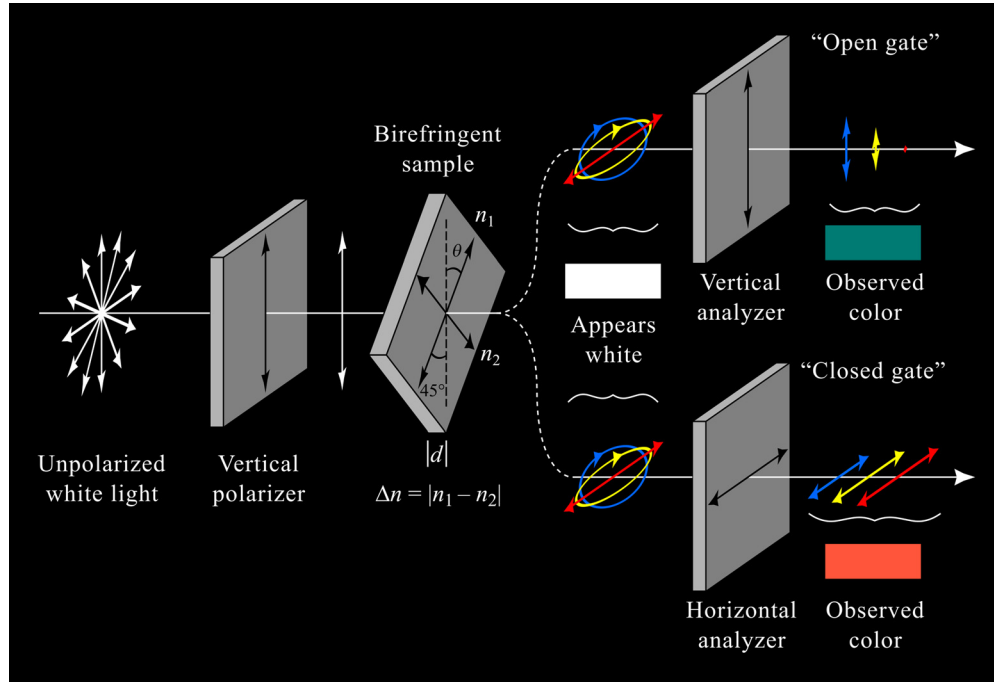


Fig. 1. (Color online) A schematic representation of the principle of polarization-filtered coloration. A vertical polarizer filters incident unpolarized white light, passing vertically-polarized light to a birefringent sample of thickness d and birefringence Δn , with its optic axis rotated by θ from the vertical. Different wavelengths within the white light experience distinct values of retardance such that their polarization states are different from each other after the sample. In the case shown here ($\theta = 45^\circ$ and $\delta = \pi$ for red light), the sample is a half-wave plate for red light, which means that linear polarization is maintained for red light, while the other colors emerge elliptically polarized, with the major axis of polarization of all wavelengths rotated by $2\theta = 90^\circ$. Despite the differing polarization across the spectrum, without further polarization filtering, the post-sample light would still appear white. Once the light passes through an analyzer, each wavelength is filtered in proportion to its polarization's projection onto the analyzer's polarization axis. For the case of an open (closed) gate, as shown on the top (bottom) path, the analyzer is a vertical (horizontal) polarizer. The open gate extinguishes the horizontally polarized red light, while the closed gate fully transmits red light. The observed color is complementary between the two gate conditions. In the example shown here, the observed color emerging from the open gate is a red-poor ocean-green, and that emerging from the closed gate is a red-laden orange-red.

and

$$T_{\text{OG}}(\lambda) = 1 - T_{\text{CG}}(\lambda). \quad (5)$$

As expected, in the closed-gate condition, all light is extinguished when the birefringent sample is oriented along one of the polarization axes (i.e., $\theta = 0^\circ$), irrespective of the magnitude of the retardance. When the birefringent sample is oriented at $\theta = 45^\circ$ with respect to the polarization axis of the input polarizer, maximum spectral modification is observed in both the open- and closed-gate arrangements, with

$$I_{\text{CG}}(\lambda) = I_0(\lambda) \sin^2\left(\frac{\pi}{\lambda} d \Delta n\right) \quad (6)$$

and

$$I_{\text{OG}}(\lambda) = I_0(\lambda) - I_{\text{CG}}(\lambda). \quad (7)$$

III. MATERIALS AND METHODS

All that is needed to create polarization-filtered color is a birefringent sample sandwiched in a polarization gate. Most computer, tablet, and cellphone screens emit linearly polarized light and are, thus, convenient replacements of traditional polarization light boxes. Displaying a full-screen blank white image suffices for creating a diffuse polarized light source, thus obviating the need for an initial polarizer. The analyzer can simply be polarized sunglasses. Designed to eliminate horizontally polarized glare off of (angled) horizontal surfaces, polarized sunglasses are linear polarizers that transmit vertically polarized light when worn. However, rotation of the sunglasses in the plane of the screen yields an analyzer with any desired orientation.

Samples capable of providing a kaleidoscopic array of colors and patterns¹³ can be easily found in any household: transparent plastic cutlery provides a classic demonstration,^{14,15} where localized strain in the polymer structure results in differential birefringence, observable through a polarization gate. Likewise, quasi-randomly folded kitchen “cling wrap” film, gift basket film, and layered adhesive tape can form intricate images reminiscent of stained-glass windows. Figure 2 shows an example of the array of colors and patterns that can be created by layering clear packaging tape on a glass plate, as illuminated by a computer screen and photographed through polarized sunglasses.

The use of polarization sheets, or polaroids, allows for the construction of more controlled and convenient polarized light sources and analyzers than do computer screens and sunglasses. Because they can be cut to any size and easily affixed to a frame, sheet polarizers are convenient for creating artistic pieces for display. A set of polarizer sheets also make an excellent classroom demonstration kit, whether for use with electronic screen illumination or as initial polarizers for use with sunlight.¹⁶

Creating a diffuse (i.e., featureless) light source is important for optimal viewing. This is easily accomplished using a piece of kitchen parchment paper or wax paper as the first layer before the initial polarizer. These papers are more translucent than standard white printer paper, yet the latter can be used if desired. It is important that the diffusing layer be placed outside of the polarization gate (i.e., before the first

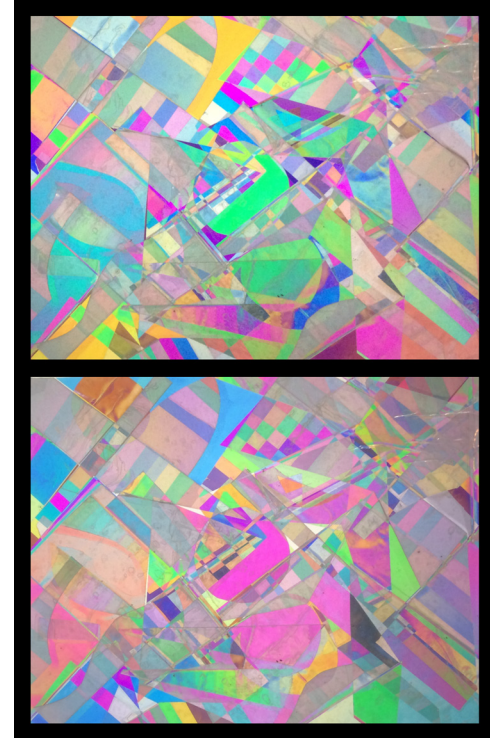


Fig. 2. (Color online) Untitled, by the author. A stained-glass-like image formed by layering adhesive tape in various orientations and thicknesses on a $20 \times 25 \text{ cm}^2$ glass plate. This image is illuminated by the diffuse polarized light of a laptop computer screen and is photographed through polarized sunglasses. In the top panel, the image is photographed through sunglasses transmitting vertical polarization, while in the bottom panel, the sunglasses are rotated by 90° to transmit horizontal polarization.

polarizer), as it can significantly depolarize the light. Figure 3 shows a work of art mounted in a simple picture frame, for window display using sunlight as an illumination source. Parchment paper is used as the backing diffusing layer. The birefringent materials comprise three different household adhesive tape products layered on picture-frame glass.

The material of choice for professional polage has traditionally been cellophane.^{9,17} In the past, most gift-wrap and household adhesive tape products were made from cellophane. Cellophane has a birefringence of approximately $\Delta n = 0.011$,^{12,18} which means that typical $15\text{--}30 \mu\text{m}$ -thick sheets can provide sufficient retardance to form a palette of colors very similar to those shown in Fig. 4 for packaging tape. Commercially, true cellophane has been supplanted by biaxially oriented polypropylene film or BOPP.¹⁹ Most BOPP packaging tape and transparent film sold as “celo”-wrap (or Sellotape) show a birefringence of $\Delta n = 0.010\text{--}0.015$ ^{12,20} and, thus, are at least as good as cellophane for polage, as demonstrated by the images in this article. Other birefringent materials from which polage can be made are polyethylene (cling wrap) and polyolefin (some high-gloss gift-wrap tapes); both of which display much smaller birefringence of $\Delta n = 0.0025\text{--}0.0045$.¹² Ultimately, the combination of materials with varying birefringence and/or thicknesses can provide a broader range of colors and color control. For example, the piece shown in Fig. 3 combines three different packaging tape products, each of which provides slightly different maximum retardance.

An artist’s palette is an important starting point for any painting, whereas an expert painter develops the ability to



Fig. 3. (Color online) Eclipse No. 1 by the author, mounted for sunlight illumination, and displayed affixed to a window. This piece was created by cutting and layering three types of adhesive tapes on the glass plate from a simple picture frame purchased at a “dollar” store. All layers fit within the plastic frame and include, from back to front: kitchen parchment paper as a diffuser, sheet polarizer with vertical polarization, glass, adhesive tape, and sheet polarizer with horizontal polarization (closed-gate configuration). The thin grey border is created with multi-layered tape arranged to allow color-independent transmission (as described in Fig. 5), thus revealing the backing diffuser layer.

mix a wider array of colors from those obtained “out of the tube,” establishing one’s palette is often an important first step. For polage, the equivalent of out of the tube colors comes from layering a given material at the maximum birefringence condition (i.e., at 45° to the gate polarization axis).¹⁷ An example of such a palette for layers of packaging tape, both for open- and closed-gate conditions, is shown in Fig. 4. It is important to establish the orthogonal birefringence directions (i.e., the optic axes) for the sample, so that the maximum birefringence condition can be found. This is most easily done by crossing the polarizers without a sample, and then inserting and rotating a single layer of the birefringent film until maximum brightness is observed at $\theta = 45^\circ$.

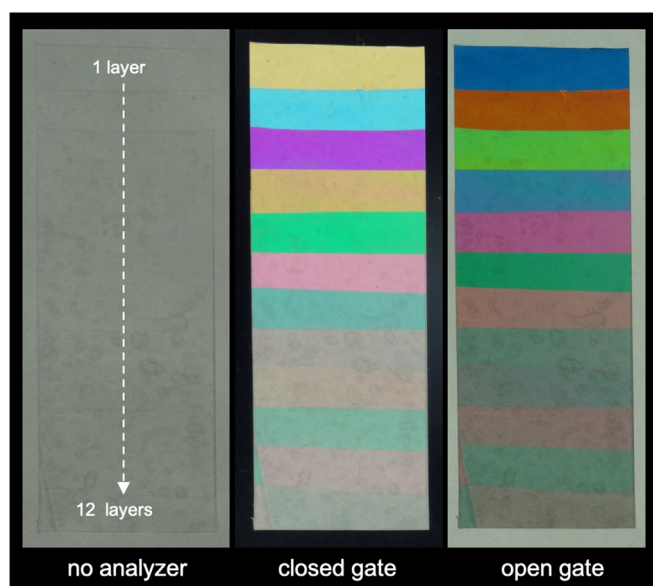


Fig. 4. (Color online) A simple palette of colors for layers of Scotch 371 packaging tape oriented at 45° to the input polarizer (i.e., at maximum birefringence) and illuminated by an incandescent halogen bulb. 1–12 overlapping layers are observed without the final polarizer (left), through crossed polarizers (middle), and through aligned polarizers (right).

For the packaging tape, the optic axes of the tape are essentially along and across the roll. This is not universally true for all samples.¹² For example, typical 19-mm gift-wrap Scotch tape gives maximum optical action when rotated by only $\theta = 10^\circ$ (not 45°), thus reflecting the fact that the optic axis is not aligned with the roll of tape in that material.

Unlike a traditional artist’s palettes because the closed-gate transmission is complementary to the open-gate transmission (as reflected by Eq. (5)), the two palettes shown in Fig. 4 cannot, in general, be intermixed in the same image. However, different films can provide unique palettes of their own, and thus, different films (and film orientations) can be used to fill gaps in available colors.

IV. PHYSICAL ATTRIBUTES OF POLARIZATION-FILTERED COLORATION

A. Layering, transmission spectra, and perceived colors

Transmission spectra provide instructive links between sample birefringence, layering, and perceived colors. Spectra from selected regions of interest in Fig. 4 are presented in Fig. 5. Consistent with Eqs. (6) and (7), the transmission is observed to be deeply modulated as a function of wavelength. For example, in the case of two layers of packaging tape viewed through a closed-gate, the 720-nm light is nearly extinguished, while the 480-nm light is fully transmitted. The overall transmitted spectrum in this case is a broad peak in the visible range, with a maximum at 515 nm, which corresponds to the color cyan, as can be seen in the center panel of Fig. 4 (and the inset in the top panel of Fig. 5). Other perceived colors are not well described by a specific central wavelength. For example, when the two layers are viewed in the open gate arrangement, the complementary spectrum is observed, with a maximum transmission at 720 nm and a nearly complete removal of the 480-nm light. This results in a color composed of strong reds and the absence of blues, which is perceived as a ruddy brown. In color theory, complementary colors are those that combine to give white light. Thus, as described conceptually by Eq. (5) and visually in

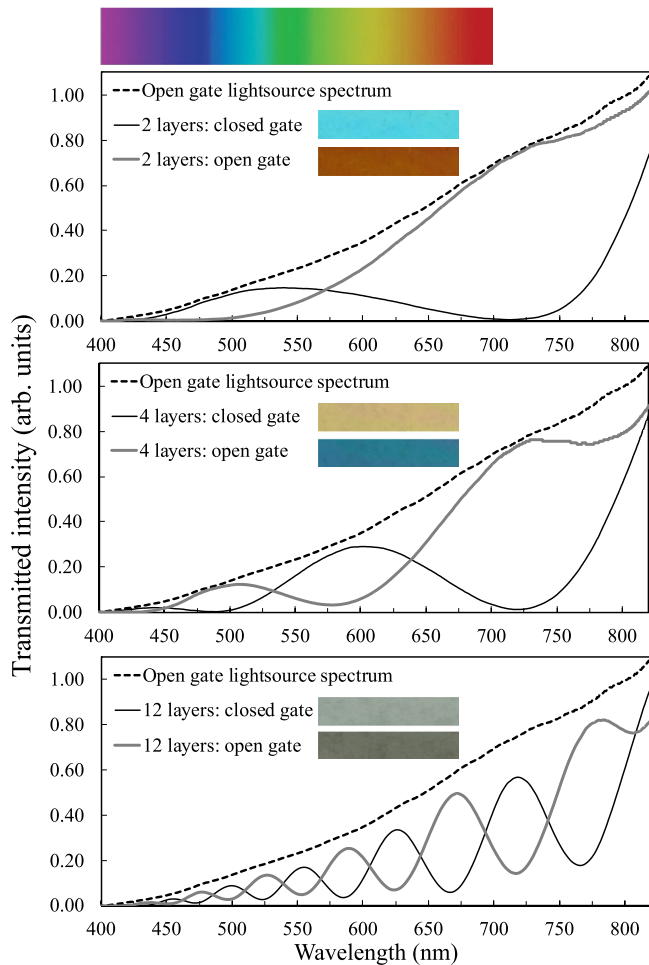


Fig. 5. (Color online) Measured transmitted spectra through layers of packaging tape in open- and closed-gate configurations, illuminated by an incandescent halogen bulb. The perceived color resulting from the transmitted spectra is reproduced as an inset in each panel and corresponds to the colors shown in Fig. 4. (Top panel) The gate-filtered spectra from two layers of tape show a first-order full-waveplate action at ~ 720 nm, observed as a minimum (maximum) transmission through a closed (open) gate. Note that the perceived closed-gate color of cyan is consistent with a broad quasi-symmetric transmission maximum near 520 nm. (Middle panel) The gate-filtered spectra from four layers of tape. (Bottom panel) The gate-filtered spectra from 12 layers of tape, in which the rapid modulation in the transmitted spectra across the visible regime is perceived as a dull grey in both open- and closed-gate configurations. Scattering from numerous interfaces and adhesive layers degrades the polarization purity of the transmitted light, leading to increasingly reduced transmission and extinction. The colors of the visible spectrum are reproduced at the top of the image, aligned to scale with the wavelength axis of the three plots, as a reference.

Fig. 4, the open and closed gate arrangements yield natural complementary color pairs.

The layering palettes in Fig. 4 show that the resulting colors become increasingly washed out as the number of stacked layers increases, eventually becoming gray. There are two main reasons for this, as can be inferred from the different spectra in Fig. 5, where the bottom panel shows the spectra from 12 layers of packaging tape. First, the spectral modulation rapidly narrows (in wavelength) with increasing sample thickness. As suggested by Eqs. (2) and (4), the transmission peaks get closer together in wavelength, and the spectrum is eventually modulated so rapidly that the perceived color is effectively an average through those modulations. This is perceived visually as a lower-intensity version

of the incident spectrum. Second, after numerous layers, the polarization purity is significantly reduced, thus diminishing the effects of birefringence and of the analyzer. This can be seen as a reduction in the modulation depth of the transmitted spectrum, as shown in the bottom panel of Fig. 5 for a 12-layer sample. In this case, the transmitted spectrum no longer shows spectral regions of high transmission and full extinction.

B. Polarization-filtered colors are not an interference phenomenon

When a birefringent material displays continuous local variation in thickness and/or birefringence, the resulting pattern of colors is highly reminiscent of those which appear in soap bubbles, oil slicks, and other thin films in which interference effects dominate.²¹ This is perhaps the reason why the phenomenon underlying polage is referred to as “interference colors.”^{9,22–24} However, as described above (and in Fig. 1), the observed colors arise from the selective filtering of spectrally variable polarization components. While the mechanism underlying the operation of some polarizers could be described in terms of interference,²⁵ the description of polage as interference phenomena is misleading. This is a possible reason why the phenomenon is not widely detailed in physics textbooks. In particular, an interference-based description implies both constructive and destructive interferences. For example, constructive interference between two beams can result in intensities that are larger than the simple sum of the (un-interfered) beam intensities. Thus, one might expect that at specific thicknesses of birefringent material, the resulting intensity at a key wavelength could exceed that of the incident illumination. However, this is not the case, and any such indications are likely measurement artefacts.⁹ In order to avoid future misconceptions and to aid in an intuitive understanding of the phenomenon, I prefer to call this phenomenon polarization-filtered coloration (PFC). While the original terminology prior to interference colors was chromatic polarization,^{1,22} I also find the latter potentially confusing because of the suggestion that the colors are distinguishable by their polarization state. Rather, the important role of the analyzer as a polarization filter, as shown in Fig. 1, should be made paramount. Hence, PFC.

C. Optical thickness and observation angle

The retardance produced by the birefringent sample depends on the distance traveled by the light in the sample. For example, when light travels at an angle through the sample, then in Eq. (1), the thickness d is replaced by the diagonal distance through the tape. Varying the observation angle, then, alters the phase retardance and, therefore, changes the observed polarization-filtered colors. This effect is well known to polage artists, who often mention the shimmering or “dynamic” aspects of their work.⁸ For large installations, the fact that viewers will see different arrays of color is taken into consideration when designing works of art.²⁶ Indeed, two observers standing next to each other may see drastically different coloration from a closely viewed work. As an example, Fig. 6 shows a sequence of photographs taken at different angles from a single work. As can be seen in the figure, any region of interest can transform in color as one’s vantage point changes. The same region can appear green,

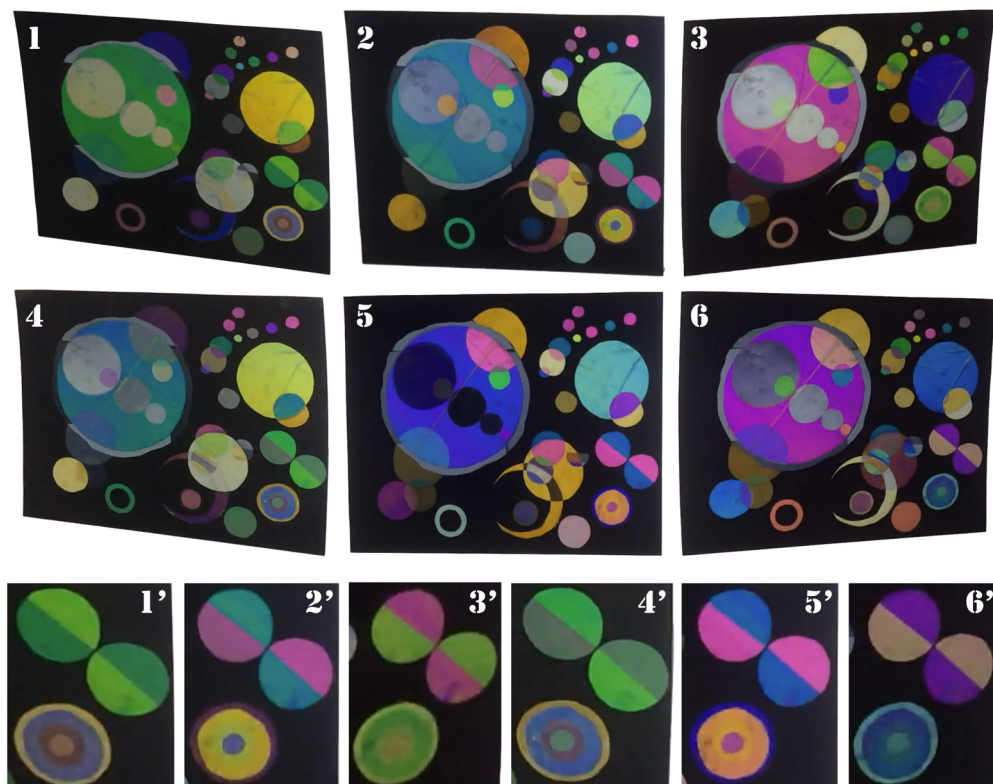


Fig. 6. (Color online) Observation angle and the effects of film thickness on transmitted color in the author's work titled "Several more circles after Kandisky." Transparent adhesive tape on the glass plate ($20 \times 25 \text{ cm}^2$), illuminated by a white-background laptop screen with crossed-polarizer and viewed from different positions about the screen: (1) viewed from above and right of the screen; (2) above the screen; (3) from above and left of the screen; (4) from the right; (5) in line with the screen; and (6) from the left. Bottom panels (1'–6') show a zoom-in region from the corresponding upper panels. Panel 5 was featured in *Optics & Photonics News*, July/August 2020.

cyan, blue, pink, or purple, with only minor changes in viewing angle.

D. Addition and subtraction of birefringence

In describing PFC, I began with the preparation of linearly polarized light but did not elucidate the range of polarization states that can emerge from the birefringent sample. The most general state of polarization—elliptical—is both conceptually and mathematically cumbersome to describe. The transmission of light through the analyzer depends both on the light's ellipticity and on the angle between the analyzer's polarization axis and the major axis of the ellipse (as hinted at in Fig. 1). Subtle changes in either parameter can lead to different transmitted spectral intensities and, thus, to different perceived colors. In general, birefringent optical elements do not commute.²⁷ This means that when light passes through a set of birefringent regions, the ordering of those regions can matter: The emerging state of polarization is different for different layering sequences. In many cases, including when observation is strictly through entirely open or closed gates, the observed colors are not perceptibly different as a function of layer ordering. However, when the analyzer is oriented in-between the open- and closed-gate conditions, the effect of layer ordering on observed color can be significant. An example of this is presented in Fig. 7, which shows a two-layer sequence of packaging and gift-wrap adhesive tapes. An ordering in which the packaging tape is the bottom layer appears dark blue, while it appears orange when the packaging tape is the top layer. Aside from the layering sequence, there is no difference between the two

regions; both comprise the same materials at the same orientation. The states of polarization emerging from the two regions in the sample are both elliptical, but likely with different major axes. When the analyzer is set at any transmission angle that is not exactly between the two major axes, the polarization filtering results in differing spectra and, thus, in different perceived colors. This is one of the principal differences between painting with colored media and painting with polarization: Whereas expertise and intuition allows

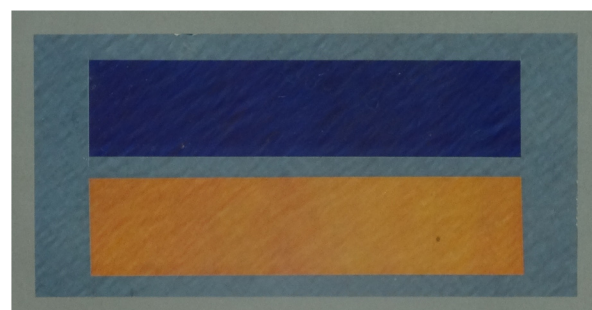


Fig. 7. (Color online) A photo of two strips of gift-wrap tape layered on/under a larger strip of packaging tape as an example of the non-commutative properties of birefringence. The layer of gift wrap tape is under the packaging tape in the top strip (appearing dark blue), while it is on top of the packaging tape in the bottom strip (appearing orange). While the two strips, thus, represent the same two layers, the sequence of birefringence changes the resultant polarization state before the analyzer and, thus, can yield different observed colors. This effect is most pronounced when the analyzer is not aligned with or orthogonal to the polarizer. In this image, the birefringence axes are at 45° to the entrance polarizer, and the analyzer is oriented at 30° to the entrance polarization.

one to predict the effects of mixing various colors to create new colors, the addition of retardances by layering media without careful attention to layer orientation and sequence leads to seemingly random outcomes. To put a fine point on it, mixing blue paint with yellow paint leads to some shade of green, but the “addition” of a blue-resulting PFC layer and a yellow-resulting PFC layer can result in any number of colors depending on the microscopic properties (thickness, birefringence, and alignment) and on the order of the layers.

A special case of predictable addition—or in this case, subtraction—arises from the understanding that birefringence axes are oriented at 90° to each other. Thus, any

rotation of the birefringent material by 90° produces the opposite retardance. When viewed through either open or closed-gate arrangements, the coloration produced by a given retardance is the same as that produced by its negative, and thus for single layers, a 90° rotation reproduces the same colors. This fact may be gleaned from the $\sin^2(\delta)$ dependence of the gate transmittance in Eqs. (3) and (5). However, if one stacks a film with another that is rotated at 90° , the result is a subtraction of the two retardances. An example of this effect is shown in Fig. 8. The figure presents an addition/subtraction grid of two different adhesive tape samples overlapping to produce three distinctly colored layers that can be designated as A, B, and $A+B$. An arrangement of layers rotated by 90° produces $-A$, $-B$, and $-(A+B)$, but the same three colors. If the tri-layer arrangements are overlapped, there are regions of complete subtraction where A and $-A$ overlap to null the retardance, and thus, no coloration is observed. Likewise, a region of $A+B$ can overlap with a region of $-B$ to produce the identical color as seen in an original A layer. This can prove useful when applied to thin layers that may be difficult to cut and place. Building up thickness via sequential layering of orthogonal orientations can produce physically thick layers with the birefringent properties of much thinner films.

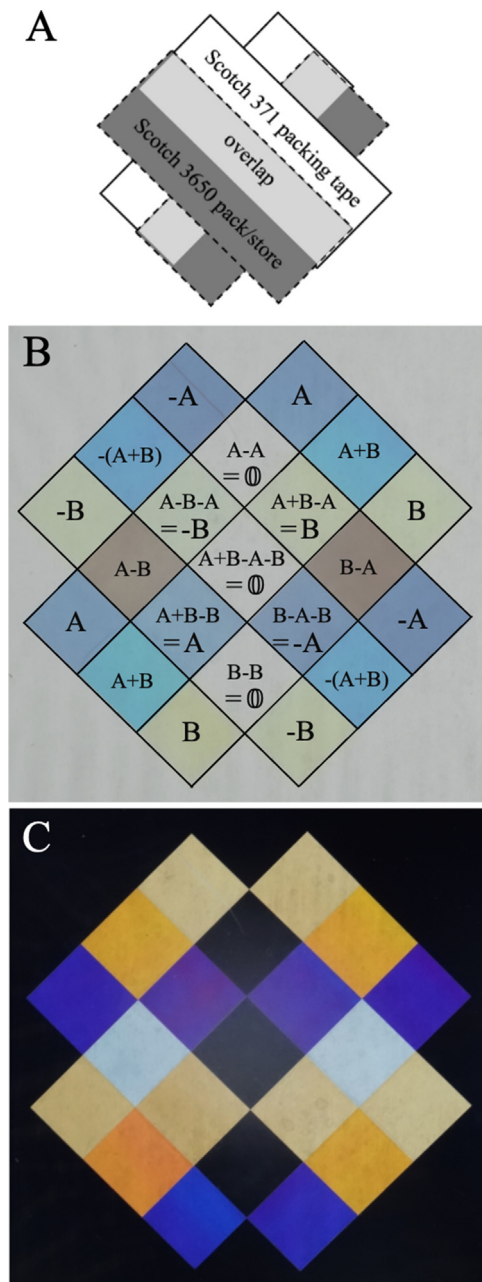


Fig. 8. (Color online) An addition/subtraction grid constructed by overlapping two different adhesive tape products in a co-aligned and orthogonal arrangements. (a) A schematic of the four-layer orientation. (b) A photograph of the arrangement sandwiched between open polarizers, with the addition and subtraction of the layers labeled in a grid. (c) A photograph of the arrangement between crossed polarizers.

V. OUTLOOK

The manipulation of birefringent films for the purpose of creating PFC images is fun and intellectually stimulating. Much of the nuanced physics of polarization, birefringence, retardance, and color theory can be observed in this accessible yet expansive endeavor. The best part is that one need not understand the scientific aspects of the phenomenon in order to appreciate, or even master, the little-known artform of painting in polarization.

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^aORCID: 0000-0001-5699-1403.

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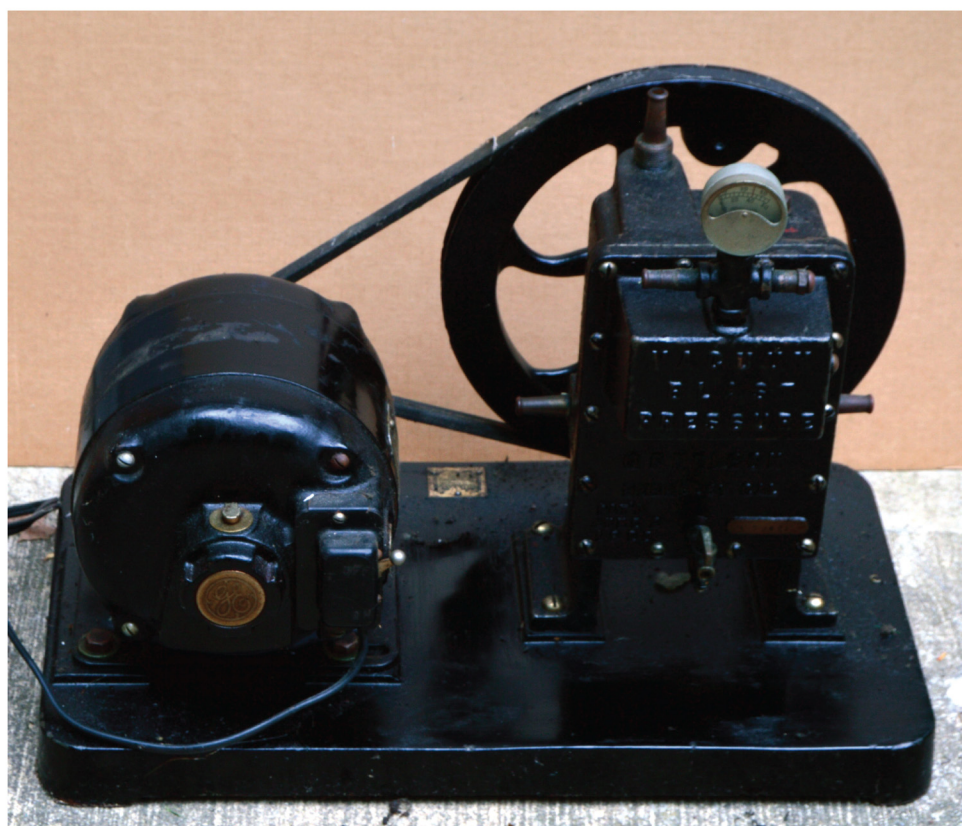
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Vacuum, Pressure and Blast Pump

This is a unique air pump. Using one set of connections, you can use it to supply a steady stream of air to a blast lamp used in glass-blowing. The gauge at the top indicates the degree of over-pressure (up to 50 pounds per square inch) that it can produce. And, as a straight rotary vacuum pump, it will go down to 0.02 mm Hg. Connecting two of these in series allows you to reach 0.00001 mm Hg, which is low enough to do experiments with discharges in gases. It is listed at \$74.50 in the 1929 catalogue of the Chicago Apparatus Company. It is in the Greenslade Collection. (Photograph and text by Thomas B. Greenslade, Jr., Kenyon College)